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General ZMP Preview Control for Bipedal Walking

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***Abstract*— An online pattern generator for bipedal walking control is designed based on the notion of ZMP preview control using the full dynamics model. The method is called the general ZMP preview control. Conventional implementation by Kajita is based on an approximated table-cart model, which neglects the actual system dynamics. This leads to erroneous ZMP tracking for real robots, even if the system tracks the generated COM pattern exactly. Numerical simulations are provided to show the advantage and character of the method.**

I. INTRODUCTION

In order to realize some behaviors for humanoid robots, such as walking, motion planners should take dynamical consequence of resulting motion into account at the same time. This is mainly due to the fact that they are supported by ground reaction forces at the supporting foot(s), which are unilateral in nature. This constraint is succinctly expressed by the notion of the zero-moment point (ZMP) [1]. Unilaterality of the contact forces is translated to the condition that the ZMP should lie within the interior of the suppoting polygon. If this condition holds, robots do not rotate about any boundary of the supporting polygon, guaranteeing that the current contact is securely preserved (as long as friction is high enough.) This is essential for stable control of humanoid robots [2].

Successful bipedal walking requires intermittent exchange of a supporting foot, while the whole body travels in such a way that the ZMP should lie within the footprint of the supporting foot(s). One can find three methods to generate walking motion. First, there are a few methods centered around the notion of rhythmic oscillators, such as CPG [3], [4]. However, it is not easy to find a successful application of the method to real human-sized biped robots. Reactive control coupled with a finite state machine forms the second class of walking generation. It sequentially exchanges the supporting foot depending on some state of the robot, e.g. the center-of-mass (COM) position relative to the current supporting foot. Pratt [5] demonstrated that this could be quite an effective method to realize human-like walking. It, however, seems that one may experience difficulty to apply his method to robots with high joint stiffness subject to high gain feedback. The third and most popular method consists of pattern-based walking [6], [7], [8]. For example, a pattern of the movement of the COM is planned based on a much simplified (but effective) dynamic model of the robot, given that footstep planning has been done.

One may conceptualize a process of pattern-based walking control by Fig. 1. Given a desired walking, such as speed and direction, walking planner generates some future footsteps, which specifies the supporting polygon for the ZMP. Based on the planned footstep, pattern generator produces the trajectory of a set of pattern variables, such as the COM. Dynamics of the system should be reflected to design a pattern generator. An aim of pattern controllers is to track the desired trajectory of the pattern variables. Joint servo coupled with the inverse kinematics [9], [10] has been applied, but full dynamic control could be used for potentially better locomotion behavior. Since there are many redundant degrees-of-freedom available, one can impose additional behavior in pattern controller. In any cases, pattern controller generates control inputs to the robot. At the same time, it can modify the ZMP position as well as foot steps to stabilize walking against the pattern tracking error, ZMP tracking error, and/or walking error.[1]

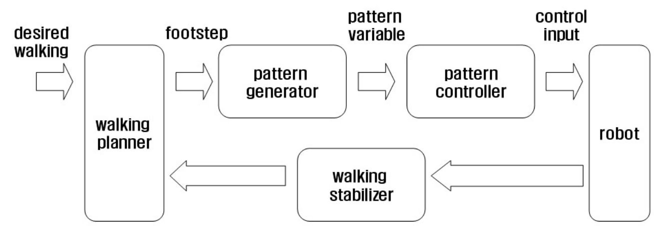


Fig. 1. Pattern-based walking control

In this article, we propose a pattern generator which generalizes the ZMP preview control developed by Kajita [8]. One of main advantages of the new method lies in the fact that the full dynamics of the robot is utilized, whereas only approximated dynamics of the table-cart model is used for pervious method. The proposed method is called the general ZMP preview control. The full dynamics can be taken into account *exactly* in terms of the pattern variables consisting of the COM and the angular momentum (about the COM) of the whole system. In original method, the COM only is the pattern variable. The content of the paper is as follows: Next section reviews the ZMP preview control and derives the general ZMP preview control. Pattern generation performance is numerically verified and compared. Then, the control performance is analyzed for both method, based on

[1]Pattern-tracking error refers to the error in tracking the pattern variable, and ZMP tracking error the error in tracking the desired ZMP. The error in terms of walking speed or position is termed the walking error.

numerical simulation of typical walking employing 12-dof biped model in Section 3.

II. Pattern Generator

A. ZMP preview with table-cart model [8]

Assume that the table-cart dynamics is valid for a general biped robot

$$p = r - T_c^2 \ddot{r}, \qquad T_c = \sqrt{\frac{z_c}{g}}, \tag{1}$$

where $p = (p_x, p_y)^T$ is the position of ZMP and $r = (x, y)^T$ is the horizontal position of the cart (equal to the COMof the whole robot). Regarding this as the output equation, we redefine a new control input u by

$$\frac{d\ddot{r}}{dt} = u. \tag{2}$$

For the state $X(t) = (r^T(t), \dot{r}^T(t), \ddot{r}^T(t))^T$, the discretized system is

$$X(k+1) = AX(k) + Bu(k) \tag{3a}$$
$$p(k) = CX(k), \tag{3b}$$

where

$$A = \begin{bmatrix} I & \Delta t I & \Delta t^2/2I \\ 0 & I & \Delta t I \\ 0 & 0 & I \end{bmatrix}$$
$$B = \begin{bmatrix} \Delta t^3/6I \\ \Delta t^2/2I \\ \Delta t I \end{bmatrix}$$
$$C = \begin{bmatrix} I & 0 & -T_c^2 I \end{bmatrix}$$

for the identity matrix I and the zero matrix 0 of size 2-by-2.

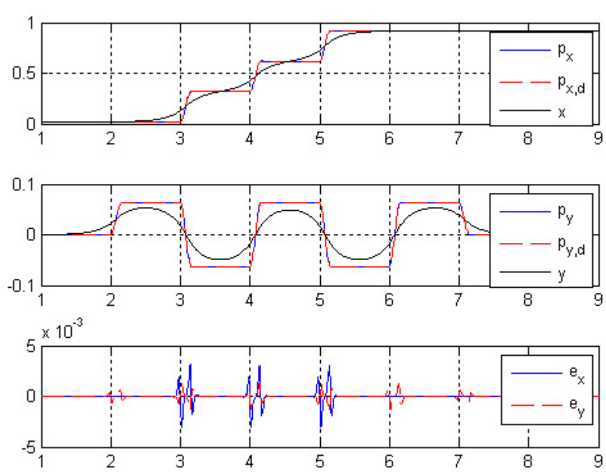


Fig. 2. Pattern generation by ZMP preview

To robustify the control [11], we add integral action by augmenting the system with the incremental control $\Delta u(k) = u(k) - u(k-1)$. Denoting the incremental state by $\Delta X(k) = X(k) - X(k-1)$, the state is augmented as $\widetilde{X}(k) = \begin{bmatrix} p(k) \\ \Delta X(k) \end{bmatrix}$. Then, the state dynamics is written as

$$\widetilde{X}(k+1) = \widetilde{A}\widetilde{X}(k) + \widetilde{B}\Delta u(k) \tag{4a}$$
$$p(k) = \widetilde{C}\widetilde{X}(k) \tag{4b}$$

where

$$\widetilde{A} = \begin{bmatrix} I & CA \\ 0 & A \end{bmatrix}, \quad \widetilde{B} = \begin{bmatrix} CB \\ B \end{bmatrix}, \quad \widetilde{C} = \begin{bmatrix} I & 0 & 0 & 0 \end{bmatrix}.$$

An optimal control problem is solved by minimizing

$$J = \sum_{i=k}^{\infty} Q_e(p_d(i) - p(i))^2 + \Delta X^T Q_x \Delta X + R\Delta u(i)^2 \tag{5}$$

which leads to [11]

$$u(k) = -K_s \sum_{i=0}^{k} (p(i) - p_d(i)) - K_x X(k) - \sum_{i=1}^{N_L} G(i) p_d(k+i). \tag{6}$$

The parameter N_L determines the preview horizon of the future desired ZMP. Experience tells that two future desired foot steps generates smooth trajectory well. The optimal gain is determined by solving the discrete algebraic Riccati equation

$$\widetilde{P} = \widetilde{A}^T\widetilde{P}\widetilde{A} - \widetilde{A}^T\widetilde{P}\widetilde{B}\left(R + \widetilde{B}^T\widetilde{P}\widetilde{B}\right)^{-1}\widetilde{B}^T\widetilde{P}\widetilde{A} + \widetilde{Q} \tag{7a}$$

where $\widetilde{Q} = \mathrm{diag}\{Q_e, Q_x\}$. Then, the optimal gain is defined by

$$\widetilde{K} = \left(R + \widetilde{B}^T\widetilde{P}\widetilde{B}\right)^{-1}\widetilde{B}^T\widetilde{P}\widetilde{A} = \begin{bmatrix} K_e & K_x \end{bmatrix}. \tag{7b}$$

The optimal preview gain is recursively computed as follows. Denote $\widetilde{A}_c = \widetilde{A} - \widetilde{B}\widetilde{K}$.

$$G(i) = \left(R + \widetilde{B}^T\widetilde{P}\widetilde{B}\right)^{-1}\widetilde{B}^T\widetilde{X}(i-1) \tag{7c}$$
$$\widetilde{X}(i) = \widetilde{A}_c^T\widetilde{X}(i-1). \tag{7d}$$

where $G(1) = -K_e$, $\widetilde{X}(1) = -\widetilde{A}_c^T\widetilde{P}\begin{bmatrix} I \\ 0 \end{bmatrix}$.

When $Q_e = I$, $Q_x = 0$ and $R = 1 \times 10^{-6} I$, the generator produces the desired motion of the COM as shown in Fig. 2. The one walking step consists of 0.85 (sec) of the single support phase, followed by 0.15 (sec) of the double support phase. The stride of a single step is $0.15(m)$. The ZMP error shown in the last row of the figures is not of particular interest, because it is computed by the approximated table-card model. Even if the pattern is perfectly reproduced by the real biped robot, the ZMP tracking error should be different from this figure, due to the dynamics discrepancy. This will be shown in the next section.

B. ZMP preview with full dynamics

We do not presume the table-cart dynamics. Let us introduce the horizontal angular momentum (HAM) of the whole system around the COM, denoted by H. Then, the *exact* ZMP equation with no external force/moment is written as [12]

$$p = r - \widetilde{T}^2 \ddot{r} + \frac{1}{M\widetilde{g}} S\dot{H}, \quad S = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \tag{8}$$

where $\widetilde{g} = g + \ddot{z}$ and $\widetilde{T} = \sqrt{z/\widetilde{g}}$. When the COM is constrained to travel at constant height z_c as in ZMP preview control, the equation reduces to

$$p = r - T_c^2 \ddot{r} + \frac{1}{Mg} S\dot{H}. \tag{9}$$

In the equation, the pattern variables include the HAM as well as the COM. It is noted that only if the HAM is perfectly suppressed, i.e. $H \equiv 0$, the table-card dynamics is the accurate model of the system dynamics.

It is worth noting that the ZMP equation (9) can be implemented exactly, as long as the total mass M is exactly estimated. It is relatively easy to measure, because it is a scalar. Now, perfect tracking of the both pattern variables guarantees perfect ZMP tracking, while for the former case it is not. This is a significant advantage over the previous ZMP preview control.

The generation algorithm is almost similar to the previous one. Let us take the state by

$$X(t) = (r^T(t), \dot{r}^T(t), H^T(t), \ddot{r}^T(t), \dot{H}^T(t))^T$$

for the pattern generator dynamics

$$\begin{bmatrix} d\ddot{r}/dt \\ d\dot{H}/dt \end{bmatrix} = \begin{bmatrix} u \\ w \end{bmatrix} = U. \tag{10}$$

For the sampling period Δt, the discretized system is

$$X(k+1) = AX(k) + BU(k) \tag{11a}$$

$$p(k) = CX(k), \tag{11b}$$

where

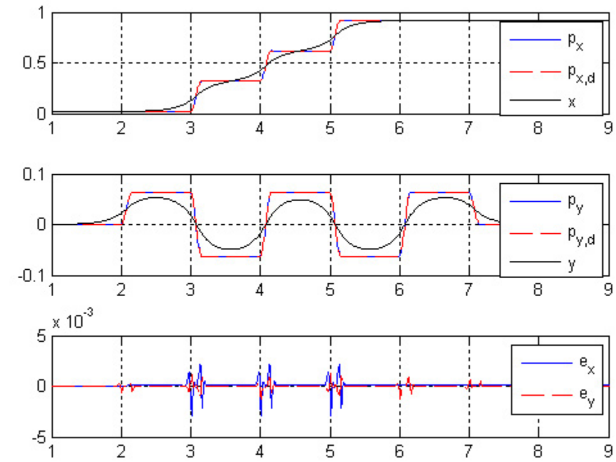
$$A = \begin{bmatrix} I & \Delta t I & 0 & \Delta t^2/2I & 0 \\ 0 & I & 0 & \Delta t I & 0 \\ 0 & 0 & I & 0 & \Delta t I \\ 0 & 0 & 0 & I & 0 \\ 0 & 0 & 0 & 0 & I \end{bmatrix}$$

$$B = \begin{bmatrix} \Delta t^3/6I & 0 \\ \Delta t^2/2I & 0 \\ 0 & \Delta t^2/2I \\ \Delta t I & 0 \\ 0 & \Delta t I \end{bmatrix}$$

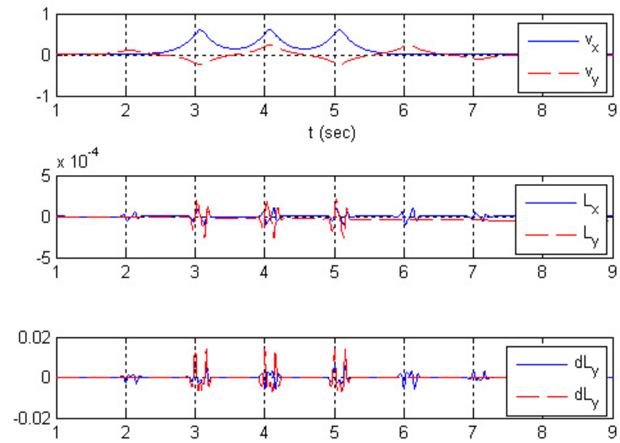
$$C = \begin{bmatrix} I & 0 & 0 & -T_c^2 I & \frac{1}{Mg} S \end{bmatrix}.$$

Then, the same procedure is applied to obtain the optimal control of the same form as (6).

We set the error and the control weighting $Q_e = I$ and $R = 1 \times 10^{-6} I$. Then, the state weighting is defined by $Q_x = \mathrm{diag}\{0, 0, \lambda_H I, 0, 0\}$, that is the angular momentum is only penalized by λ_H. For the value of $\lambda_H = 10$, the generator produces the following results shown in Fig. 3. One can reduce the angular momentum consumption by enlarging the weight λ_H. Note that when $\lambda_H = 0$ the system is not stabilizable.



(a) generated COM trajectory



(b) generated momentum trajectory

Fig. 3. Pattern generation by general ZMP preview control

III. Pattern Control

Since biped robots are free-floating articulated multibody systems, the equations of motion can be expressed as [13]

$$W\widetilde{F} + L\tau = M \begin{bmatrix} \dot{V} \\ \ddot{q} \end{bmatrix} + C \begin{bmatrix} V \\ \dot{q} \end{bmatrix}, \tag{12}$$

where M is the system inertia, C the system bias matrix, W the wrench influence matrix, and L is the torque influence matrix with V denoting the base body twist, $\dot{q}$ the joint velocity vector, $\widetilde{F}$ the aggregation of each body wrench including gravitational one, τ the joint torque vector. In presence of a single support foot, the equation reduces to [2]

$$W_e \widetilde{F} + \tau = M_e \ddot{q} + C_e \dot{q}, \tag{13}$$

as long as the support foot keeps secure contact. For the case of double support phase, the system forms a closed-loop, whose equations of motion are expressed by

$$W_r \widetilde{F} + L_r \tau = M_r \dot{y} + C_t y \tag{14}$$

for an independent variable $y = Q\dot{q}$ defined by a null space bases matrix of the constraint.

The equations such as (13) and (14) can be used to design an inverse dynamic control to track the generated motion of the pattern variables. In walking control, pattern variables are functions of joint position and/or velocity. For single support phase, the velocity of pattern variables denoted by ξ is expressed by

$$\xi = J(q)\dot{q}$$

for a rectangular Jacobian matrix $J(q) \in \mathbb{R}^{m\times n}$. The pattern variables can be controlled by generating the reference joint acceleration $\ddot{q}_{\mathrm{ref}}$ by

$$\ddot{q}_{\mathrm{ref}} = J^{-1}(q)(\dot{\xi}_{\mathrm{ref}} - \dot{J}(q,\dot{q})\dot{q})$$

in the case where the Jacobian is a nonsingular square matrix. Otherwise, redundancy resolution schemes, e.g. [14], have to be applied. To track the desired pattern ξ_{des}, $\dot{\xi}_{\mathrm{ref}}$ is properly defined , e.g. by $\dot{\xi}_{\mathrm{ref}} = \dot{\xi}_{\mathrm{des}} + k_v(\xi_{\mathrm{des}} - \xi) + k_p \int (\xi_{\mathrm{des}} - \xi)dt$. For double-support phase, similar approach is applied to compute $\dot{y}_{\mathrm{ref}}$. Then, the pattern variables can be tracked in an exponentially stable manner, provided that the dynamics parameters are correct.

A. ZMP preview control

When the desired pattern regarding the COM has been applied to the simulation[2] of an experimental biped platform, shown in Fig. 4, it produces the result summarized in Fig. 5. The actual ZMP trajectory deviates much from the desired one (see the last figure), although the desired COM trajectory is almost perfectly tracked. The robot suffers from lack of stability margin during walking, as shown in Fig. 6 which illustrates the ZMP trajectory (darker line) and the COM trajectory (lighter line) in relation with the footprints. Small disturbances may destabilize walking, without the help of a strong walking stabilizer.[3]

B. General ZMP preview control

For the general ZMP preview control, the desired HAM should be controlled additionally. The control results are summarized in Figs. 7 and 8. Significant improvement in ZMP tracking performance can be observed in Fig. 8 as well as the last figure of Fig. 7 (a). As one can expect, the system can enjoy quite broad stability margin during walking, contrary to the previous result.

[2]The hip orientation is regulated at the same time.

[3]As a matter of fact, Kajita [8] remedies this problem by applying a dynamics filter based on the ZMP preview control.

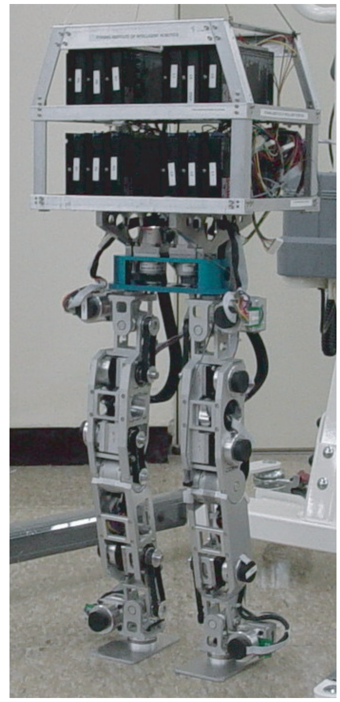


Fig. 4. Experimental biped platform (12 dof, $M = 16.424$ (Kg))

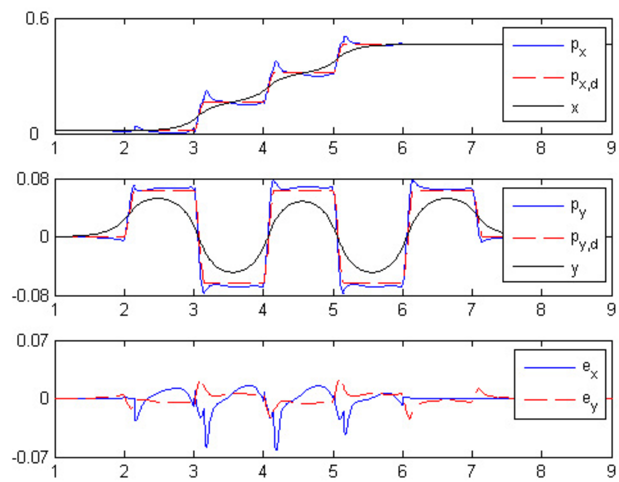


Fig. 5. Actual ZMP by ZMP preview control

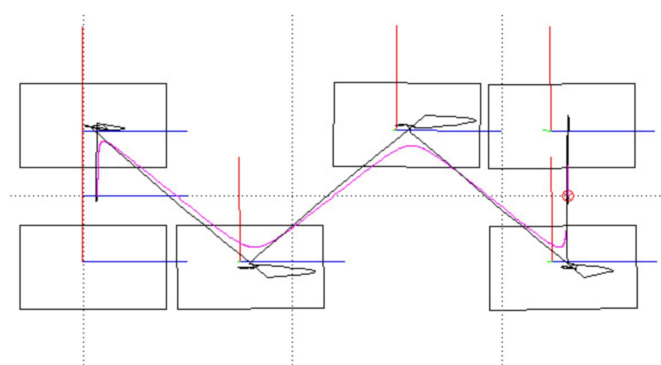
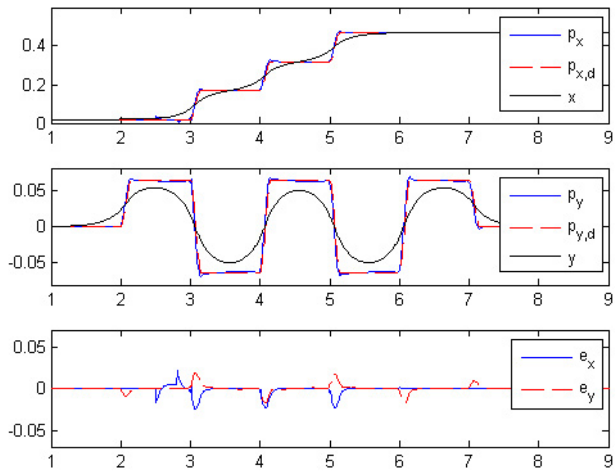
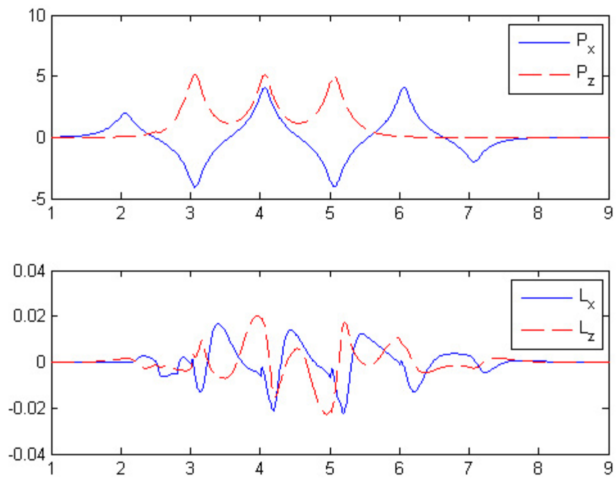


Fig. 6. Footprints of ZMP preview control

As a matter of fact, the system experiences unexpected external horizontal forces at the hip from 2.5 to 2.8 (sec) of magnitude $5(N)$. The ZMP error is relatively larger than normal, as can be seen in the last figure of Fig. 7 (a). It is worth noting that since the pattern generator itself has ZMP error integral feedback, embodied as the first term of (6), the ZMP error is eliminated autonomously.



(a) generated ZMP trajectory



(b) generated momentum trajectory

Fig. 7. Result of general ZMP preview control

C. ZMP tracking performance

To demonstrate online ZMP tracking performance of the proposed method, we simulate the robot subject to rather extreme disturbance situation. The hip is disturbed by horizontal forces of the form

$$\begin{bmatrix} f_x(t) \\ f_y(t) \end{bmatrix} = [8u_x(t), 8u_y(t)]$$

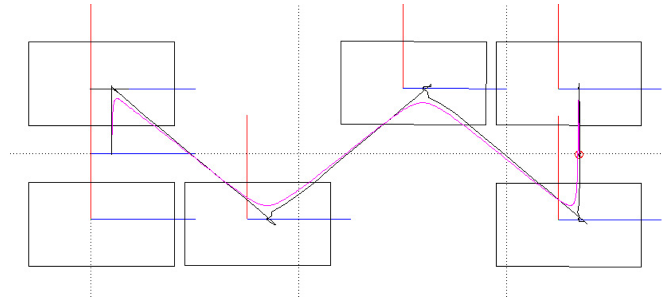


Fig. 8. Footprints of general ZMP preview control

with probability of 50%, where $u_x(t), u_y(t)$ belongs to the uniformly distributed random number belonging to $[-1, 1]$.[4] The simulation result is summarized in Figs. 9 and 10. The simulation shows that the ZMP is kept securely within the support polygon during whole walking. As shown in the last figure of Fig. 9 (a), the ZMP tracking error is bounded about $\pm 0.03(m)$. Note that the HAM pattern generated online (of small magnitude) works effectively to stabilize walking.

IV. CONCLUSION

General ZMP preview control taking into account of the full dynamics of biped robots have been proposed as a pattern generator and a walking stabilizer at the same time for bipedal walking. It generates the desired motion of pattern variables consisting of the COM and the HAM in online, given a few future footstep has been planned (two in this article). Pattern generation can be done with the exact system dynamics, as long as the total mass is exactly available. At the same time, the general ZMP preview control works fine as a walking stabilizer, since the integral error of the ZMP tracking error is fedback through optimally designed gain matrix.

There still remains some problem about the proposed general ZMP preview control. It is not a simple matter to determine the appropriate weight matrices such as R, Q_e and Q_x. In particular, there are two factors related with the center-of-mass motion and the angular momentum. It can be said that they put a guideline of trade-off between two motions. Next, the body orientation may not be properly regulated due to angular momentum specification in case of insufficient degrees-of-freedom.

ACKNOWLEDGMENTS

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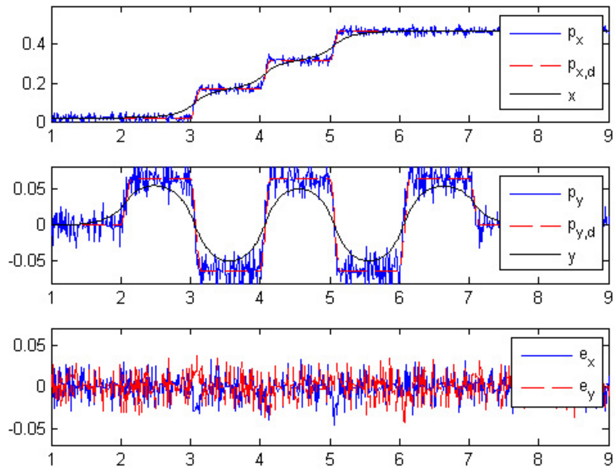
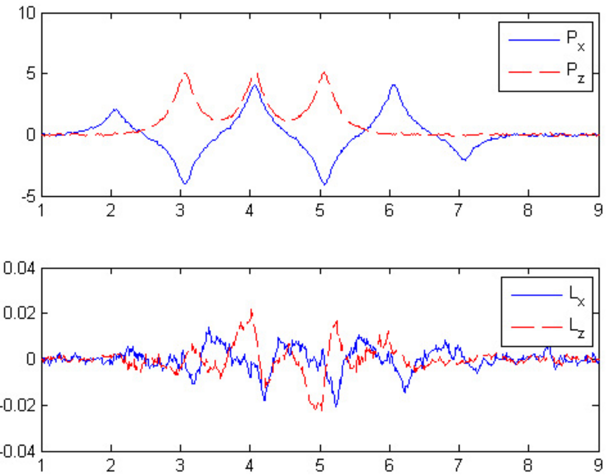
[4] In Matlab, it is implemented as:

```
if rand() > 0.5
  f_x = 8*(-1 + 2*rand());
  f_y = 8*(-1 + 2*rand());
end.
```



(a) generated ZMP trajectory



(b) generated momentum trajectory

Fig. 9. General ZMP preview control subject to extreme disturbances

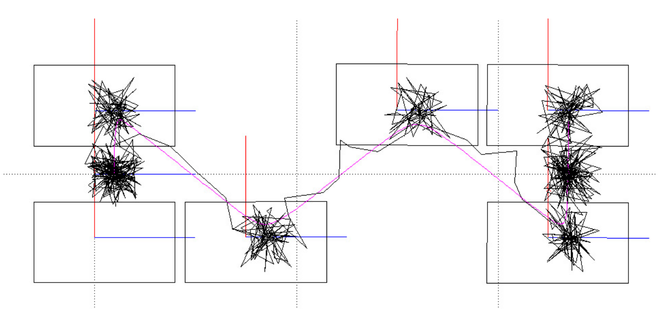


Fig. 10. Footprint of general ZMP preview control subject to extreme disturbances

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